(7 Pages) **Reg. No. : ....**

Code No. : 20592 E **Sub. Code : SEMA 6 E**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Sixth Semester

Mathematics

Major Elective — CODING THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The ___________ of the word is the number of digits in the word.

   (a) length (b) code

   (c) decode (d) none

2. The word that belong to a given code with be called ___________ code.

   (a) binary (b) block

   (c) perfect (d) none

3. Any set of vectors containing zero is ___________

   (a) Linearly independent

   (b) Linearly dependent

   (c) Null space

   (d) None

4. A code C is a ___________ code if $u+v$ is a word in C whenever u and v are in C.

   (a) linear (b) equal

   (c) dual (d) none

5. A matrix H is a parity-check matrix for some ___________ code C iff the columns of H are linearly independent.

   (a) linear (b) symmetric

   (c) perfect (d) none

6. If C is a linear code of length n and dimension k, the rank of the generator matrix is ___________

   (a) k (b) n

   (c) $k+n$ (d) kn

7. The Golay code C_{23} is a ___________ code.

   (a) error (b) imperfect

   (c) perfect (d) none

8. In the r^{th} order Reed-Muller code $RM(r, m)$, length $n =$ ___________

(a) 2^r (b) 2^m

(c) 2^{r-1} (d) None

9. The extended code C_{23}^* is indeed of ___________

(a) C_{23} (b) C_{24}

(c) C_{24}^* (d) none

10. Hamming codes are ___________ single error correcting codes.

(a) linear (b) perfect

(c) dual (d) none

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write the basic assumption about the channel.

Or

(b) Suppose we have a BSC with $\frac{1}{2} < p < 1$. Let v_1 and v_2 be code words and w a word. Each of length b. Suppose that v_1 and w disagree in d_1 positions and v_2 and w disagree in d_2 positions. Then prove that $\varphi_p(v_1, w) \le \varphi_p(v_2, w)$ iff $d_1 \ge d_2$.

12. (a) Show that $C = \{0000, 1100, 0011, 1111\}$ is a linear code and that its distance is $d = 2$.

Or

(b) Find the different number of basis for the code $C = \langle S \rangle$, where $S = \{010, 011, 111\}$.

13. (a) Find a systematic code C' equivalent to the given code C. Check that C and C' have the same length, dimension and distance.

(i) $C = \{00000, 10110, 10101, 00011\}$

(ii) $C = \{00000, 11100, 00111, 11011\}$.

Or

(b) List the cosets of the linear code C with the generator matrix $G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$.

14. (a) For any (n, k, d) linear code, prove that $d - 1 \leq n - k$.

Or

(b) What is a lower an upper bound on the size or the dimension of a code with $n = 9$ and $d = 5$?

15. (a) Find the remainder and the quotient dividing $f(x) = 1 + x^2 + x^6 + x^9 + x^{11}$ by $g(x) = 1 + x^2 + x^5$.

Or

(b) Let $a \leftrightarrow a(x)$, $b \leftrightarrow b(x)$ and $b' \leftrightarrow b'(x) = x^n b\left(x^{-1}\right)$ mod $1 + x^n$. Prove that $a(x)b(x)$ mod $1 + x^n = 0$ if and only if $\pi^k(a) \cdot b' = 0$ for $k = 0, 1, 2, \ldots, n-1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) Find the length of the information rate of a code $C_2 = \{000000, 010101, 101010, 111111\}$.

(ii) Define Information rate, Channel; Symmetric.

Or

(b) Explain how to convert a channel with $0 \le p \le \frac{1}{2}$ into a channel with $\frac{1}{2} \le p < 1$. What can be said about a channel with $p = \frac{1}{2}$?

17. (a) Find a basic B for the code $C = \langle S \rangle$, where $S = \{1010, 0101, 1111\}$ and a basic $B^{\perp}$ for the dual code $C^{\perp}$.

Or

(b) Let b_n be the number of different bases for K^n, verify the entries in the following table.

n	1	2	3	4	5	6
b_n	1	3	28	840	83328	27998208

18. (a) Lit the cosets of the linear code C = {0000, 1011, 1010, 1110} and also find the number of different cosets.

Or

(b) Find the parity check matrices from the following :

(i) C = {0000, 1110, 0111, 1001}

(ii) C = {0000, 1001, 0110, 1111}

(iii) C = {00000, 11110, 01111, 10001}.

19. (a) Show that the weight of any word in C_{24} is a multiple of 4.

Or

(b) Write an algorithm for IMLD for C_{24}. Further decode w = 001001001101, 101000101000 given that the syndrome s = 100000000001.

20. (a) Let C be a cyclic code of length n and let $g(x)$ be the generator polynomial. If $n-k$ = degree $(g(x))$, prove that

(i) The codewords corresponding to $g(x)$, $xg(x), \ldots, x^{k-1}g(x)$ are a basis for C.

(ii) $c(x) \in C$ if and only if $c(x) = a(x)g(x)$ for some polynomial $a(x)$ with degree $(a(x)) < k$.

Or

(b) The generator polynomial $g(x)$ for the smallest cyclic code of length n containing the word v (polynomial $v(x)$) is the greatest common divisor of $v(x)$ and $1 + x^n$.